

DAY OF THE MOCK EXAM















































QUESTION 2 ON PAGE 15

1 Question

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\begin{cases}
a_1 = -18 \\
a_{n+1} = 3a_n + 40n - 22 & (n = 1, 2, 3...)
\end{cases}
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The above defines the sequence (a_n) . 1. Find a_2 .

- 2. Let $b_n = a_{n+1} a_n$, for n = 1, 2, 3, ... Express b_{n+1} in terms of b_n .
- Find b_n, and thus a_n, explicitly.
- Find the value(s) of n such that a_n is a multiple of 4.
 - That the total (b) of 17 back that on a 2 of materials of 4

Solution

1. $a_2 = a_{1+1} = 3 \cdot a_1 + 40 \cdot 1 - 22 = -3 \cdot 18 + 40 - 22 = -36$

2. We are given $b_n = a_{n+1} - a_n$, so $b_n = 2a_n + 40n - 22$. Now $b_{n+1} = 2a_{n+1} + 40(n+1) - 22 = 2(3a_n + 40n - 22) + 40n + 40 - 22$

Now $b_{n+1} = 2a_{n+1} + 40(n+1) - 22 = 2(3a_n + 40n - 22) + 40n + 40 - 22$ = $6a_n + 80n - 44 + 40n + 40 - 22$. This equals $6a_n + 120n - 26 = 3b_n + 40$.

3. First let us determine $b_1 = a_2 - a_1 = -18$.

Now for n > 1 we have $b_n = 40(1 + 3 + ... + 3^{n-2}) - 18 \cdot 3^{n-1}$. Recall that as the sum of a geometric progression $(1 + 3 + ... + 3^{n-2}) = (3^{n-1} - 1)/(3 - 1)$.

So we have $b_n = 20(3^{n-1} - 1) - 18 \cdot 3^{n-1} = 2 \cdot 3^{n-1} - 20$.

Since $b_n = 2a_n + 40n - 22$, we have $a_n = (b_n - 40n + 22)/2 = (2 \cdot 3^{n-1} - 20 - 40n + 22)/2 = 3^{n-1} + 1 - 20n$.

Check this for n = 2: $a_2 = 3 + 1 - 20 \cdot 2 = -36$. 4. We would like $3^{n-1} + 1 - 20n \equiv 0 \mod 4$.

We may disregard the 20n as 20 is already a multiple of 4, so we just need $3^{n-1} + 1 \equiv (-1)^{n-1} + 1 \equiv 0 \mod 4$, which is the case for every even n. For every odd n we have a remainder of $1 + 1 = 2 \mod 4$.

